VOGAN DIAGRAMS OF AFFINE UNTWISTED KAC-MOODY SUPERALGEBRAS

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Abstract. This article classifies the Vogan diagram of the affine untwisted Kac Moody superalgebras.

1. Introduction

Real forms of Lie superalgebras have a growing application in superstring theory, M-theory and other branches of theoretical physics. Magic triangle of M-theory by Satake diagram has been obainted by [13]. Similarly supergravity theory can be obatined by Vogan diagrams. Symmetric spaces with the connection of real form of affine Kac-Moody algebras already studied by Vogan diagrams. Our future work will be in exploring Symetric superspaces of affine Kac-Moody superalgebras using Vogan diagram.

The last two decades shows a gradual advancement in classification of real form of semisimple Lie algebras to Lie superalgebras by Satake diagrams and Vogan diagrams. Splits Cartan subalgebra based on Satake diagram where as Vogan diagram based on maximally compact Cartan subalgebra. Batra developed the Vogan diagram of affine untwisted kac-Moody algebras [2, 1]. Here we extend the notion to superalgebra case.

If $\mathfrak{g}$ is a complex semisimple Lie algebra with Killing form B and Dynkin diagram D, its real forms $\mathfrak{g}_{\mathbb{R}} \subset \mathfrak{g}$ can be characterized by the Cartan involutions

$$\theta : \mathfrak{g}_{\mathbb{R}} \to \mathfrak{g}_{\mathbb{R}}$$

The bilinear form $B(.,\theta)$ is symmetric negative definite. The Vogan diagram denoted by (p, d), where d is a diagram involution on D and p is a painting on the vertices fixed by d. It is extended to Vogan superdiagrams on its extended Dynkin diagram[3]. Here we extend the theory to affine untwisted Kac Moody superalgebras. The future version of the article will also contain the Vogan diagram of twisted Kac-Moody superalgebras.

2. Cartan Involution and Invariant bilinear form

An involution θ of a real semisimple Lie algebra $\mathfrak{g}_0$ such that symmetric bilinear form

$$B_\theta(X, Y) = -B(X, \theta Y)$$

is strictly positive definite is called a Cartan involution.

For Contragradient Lie superalgebras there exist a supersymmetric nondegenerate invariant bilinear form on it and defined in [3] as

$$B_\theta(X, Y) = B(\theta X, \theta Y)$$

Let $\mathcal{G}^1$ be a complex affine Kac-Moody superalgebra. The uniqueness of B is extended to $\mathcal{G}^1$. The killing form is unique by when restricted to $\mathcal{G}_0$

An involution θ for affine Kac-Moody superalgebras is defined by taking identity on t^m

$$\theta(t^m \otimes x) = t^m \otimes \theta(x)$$

$$\theta(c) = c$$

and

$$\theta(d) = d$$

We say a real form has Cartan automorphism θ if B restrict to the Killing form on $t^m \otimes X$ where $X \in \mathcal{G}_0$ and B_θ is symmetric negative definite on $\mathcal{G}^1$. A bilinear supersymmetric invariant form $B^1(,)$ can be set up on $\mathcal{G}^1$ by the definitions

$$B^{(1)}(t^j \otimes X, t^k \otimes Y) = \delta^{j+k} B(X,Y)$$

$$B^{(1)}(t^j \otimes X, C) = 0$$

$$B^{(1)}(t^j \otimes X, D) = 0$$

$$B^{(1)}(C,C) = 0$$

$$B^{(1)}(C,D) = 1$$

$$B^{(1)}(D,D) = 0$$

Proposition 2.1. *Let $\theta \in aut_{2,4}(\mathcal{G}^1)$. There exists a real form $\mathcal{G}^1_{\mathbb{R}}$ such that θ restricts to a Cartan automorphism on $\mathcal{G}^1_{\mathbb{R}}$.*

Proof. Since θ is an $\mathcal{G}^1$ automorphism, it preserves B. namely

$$B(X,Y) = B(\theta X, \theta Y)$$

$B_\theta(X,Y) = B_\theta(Y,X)$, $B_\theta(X,Y) = B_\theta(\theta X, \theta Y)$, $B_\theta(X.\theta X) = 0$

$$B_\theta(X \otimes t^m, Y \otimes t^n) = B_\theta(Y \otimes t^n, X \otimes t^m) =$$

$$B(X \otimes t^m, Y \otimes t^n) = t^{m+n} B(X,Y)$$

for all $X, Y \in \mathcal{G}_0$

$$B(K, X \otimes t^k) = B(D, X \otimes t^k) = B(D,D) = B(K,K) = 0$$

For $z \in L(t,t^{-1}) \otimes \mathcal{G}_0$ and $X, Y \in L(t,t^{-1}) \otimes \mathcal{G}_1$

$$B_\theta(X,[Z,Y]) = B(X,[\theta Z, \theta Y]) = -B_\theta(X,[\theta Z, \theta Y])$$

$$B_\theta(X,[Z,Y]) = 0$$

$\forall\ X \in \mathbb{C}c$ or $\mathbb{C}d$

$\mathcal{G}^{(1)}_{\mathbb{R}} \simeq \mathcal{G}^{(1)}_{\overline{0}\mathbb{R}} \simeq \mathcal{G}_{\overline{0}\mathbb{R}}$. The above three real forms are isomorphic. So the Cartan decomposition of $\mathcal{G}^{(1)}_{\mathbb{R}}$ are isomorphic

to $\mathcal{G}_{\overline{0}}$.

$\mathcal{G}_{\overline{0}} = \mathfrak{k}_0 \oplus \mathfrak{p}_0$

$B_\theta(X,[Z,Y]) = \begin{cases} -B_\theta([Z,X],Y) & \text{if } Z \in \mathfrak{k}_0 \\ B_\theta([Z,X],Y) & \text{if } Z \in \mathfrak{p}_0 \end{cases}$

We say that a real form of $\mathcal{G}$ has Cartan automorphism $\theta \in \mathrm{aut}_{2,4}(\mathcal{G})$ if B restricts to the Killing form on $\mathcal{G}_0$ and B_θ is symmetric negative definite on $\mathcal{G}_{\mathbb{R}}$. $B_\theta(X_i, X_j) = \delta_{ij}$. It follows that B_θ negative definite on $L(t,t^{-1}) \otimes \mathcal{G}^{(1)}$. By B_θ is symmetric bilinear form on L_1 $\{1 \otimes X_1, 1 \otimes X_2, \cdots, d\}$. So it is conclude that θ is a Cartan automorphism on $\mathcal{G}^{(1)}$. $\square$

3. Vogan diagram

A root is real if it takes on real values on $\mathfrak{h}_0$,(i.e., vanishes on $\mathfrak{a}_0$) imaginary if it takes on purely imaginary values on $\mathfrak{h}_0$ (i.e., vanishes on $\mathfrak{a}_0$) and complex otherwise. A θ stable Cartan subalgebra $\mathfrak{h}_0 = \mathfrak{k}_0 \oplus \mathfrak{a}_0$ is maximally compact if its compact dimension is as large as possible, maximally noncompact if its noncompact dimensional is as large as possible. An imaginary root α compact if $\mathfrak{g}_\alpha \subseteq \mathfrak{k}$, noncompact if $\mathfrak{g}_\alpha \subseteq \mathfrak{p}$ Let $\mathfrak{g}_0$ be a real semisimple Lie algebra, Let $\mathfrak{g}$ be its complexification, let θ be a Cartan involution, let $\mathfrak{g}_0 = \mathfrak{k}_0 \oplus \mathfrak{p}_0$ be the corresponding Cartan decomposition A maximally compact θ stable Cartan subalgebra $\mathfrak{h}_0 = \mathfrak{k}_0 \oplus \mathfrak{p}_0$ of $\mathfrak{g}_0$ with complexification $\mathfrak{h} = \mathfrak{k} \oplus \mathfrak{p}$ and we let $\triangle = \triangle(\mathfrak{g}, \mathfrak{h})$ be the set of roots. Choose a positive system $\triangle^+$ for $\triangle$ that takes $i\mathfrak{t}_0$ before $\mathfrak{a}$. $\theta(\triangle^+) = \triangle^+$
$\theta(\mathfrak{h}_0) = \mathfrak{k}_0 \oplus (-1)\mathfrak{p}_0$. Therefore θ permutes the simple roots. It must fix the simple roots that are imaginary and permute in 2-cycles the simple roots that are complex. By the Vogan diagram of the triple $(\mathfrak{g}_0, \mathfrak{h}_0, \triangle^+))$., we mean the Dynkin diagram of $\triangle^+$ with the 2 element orbits under θ so labeled and with the 1-element orbits painted or not, according as the corresponding imaginary simple root is noncompact or compact.

The uniqueness of Cartan automorphism from Dynkin diagram of $\mathcal{G}_0$ to $\mathcal{G}^{(1)}$ proved in [3]. This gives a straightforward proof of the above theory to affine untwisted Kac-Moody superalgebras cases with the addtion of canonical central element K and derivation D.

Definition 3.1. *A Vogan diagram (p, d) on D of $\mathcal{G}^{(1)}$ and one of the following holds:*

(i) *θ fixes grey vertices*
(ii) *θ interchange grey vertices and $\sum_S a_\alpha$ is odd.*
(iii) *$\sum_S a_\alpha$ is odd*

Proposition 3.2. *Let $\mathcal{G}_\mathbb{R}$ be a real form, with Cartan involution $\theta \in$inv$(\mathcal{G}_\mathbb{R})$ and Vogan diagram (p, d) of D_0. The following are equivalent*

(i) *θ extend to aut$_{2,4}\mathcal{G}^{(1)}$.*
(ii) *$(\mathcal{G}_{\bar{0}\mathbb{R}})$ extend to a real form of $\mathcal{G}^{(1)}$.*
(iii) *(p, d) extend to a Vogan diagram on D*

Proof. Let S be the $d-$ orbits of vertices defined by [4]

$$S =$$

{vertices painted by p}

$$\cup$$

{white and adjacent 2-element d-orbits}

$$\cup$$

{grey and non adjacent 2-element d-orbits}

Let D be the Dynkin diagram of $\mathcal{G}^{(1)})$ of simple root system $\Phi \cup \phi(\Phi$ simple root system with ϕ lowest root) with $D = D_{\bar{0}} + D_{\bar{1}}$, where $D_{\bar{0}}$ and $D_{\bar{1}}$ are respectively the white and grey vertices. The numerical label of the diagram shows $\sum_{\alpha \in D_{\bar{1}}} = 2$ has either two grey vertices with label 1 or one grey vertex with label 2.

(i) $D_{\bar{1}} = \{\gamma, \delta\}$ so the labelling of the odd vertices are 1.
(ii) $D_{\bar{1}} = \{\gamma\}$ so labelling is 2 ($a_\alpha = 2$) on odd vertex.

$\theta \in$inv$(\mathcal{G}_\mathbb{R})$; θ permutes the weightspaces $L(t, t^{-1}) \otimes \mathcal{G}_{\bar{1}}$ The rest part of proof of the proposition is followed the proof of the propostion 2.2 of [3] □

4. Affine Kac-Moody superalgebras

Let a finite and countable set $I = \{1, \ldots, r\}$ with $\tau \subset I$. To a given generalized Cartan matrix A and subset τ, there exist a Lie superalgebra $\bar{\mathcal{G}}(A, \tau)$ with the following set of relations

$$\begin{aligned}
&[h_i, h_j] = 0 \\
&[e_i, f_j] = \delta_{ij} h_i \\
&[h_i, e_j] = a_{ij} e_j \\
&[h_i, f_j] = -a_{ij} f_j \\
&deg(h_i) = deg(f_i) = \bar{1} if\ i \in \tau \\
&deg(h_i) = deg(f_i) = \bar{0} if\ i \notin \tau
\end{aligned}$$

Let $e_{ij} = (ade_i)^{1-\frac{2a_{ij}}{a_{ii}}} e_j$ and $f_{ij} = (adf_i)^{1-\frac{2a_{ij}}{a_{ii}}} f_j$

We have the triangular decomposition of

$$\bar{\mathcal{G}}(A, \tau) = N^-_{f_i} \oplus H_{h_i} \oplus N^+_{f_i}$$

Let the ideal of $N^-_{f_i}$ generated by $[f_i, f_j]$ is R^- and the ideal generated $[e_i, e_j]$ by $N^+_{f_i}$ is R^+ such that $a_{ij} = 0$ and all the f_{ij} and $e_{ij}$for the former and later respectivily. $R = R^+ \otimes R^-$ is an ideal of $\mathcal{G}(A, \tau)$[11]. The quotient $\bar{\mathcal{G}}(A, \tau)/R = \mathcal{G}(A, \tau)$ is called a generalised Kac-Moody superalgebra.

(a) $\mathcal{G}^{(1)}$ is an affine Kac-Moody superalgebra if A is indecomposable.

(b) There exists a vector $(a_i)_{i=1}^{m+n}$, with a_i all positive such that $A(a_i)_{i=1}^{m+n}$=0. Then A is called Cartan matrix of affine type. The Affine superalgebra associted with a generalized Cartan matrix of type $X^1(m, n)$ is called untwised affine Kac-Moody superalgebra.

4.1. Dynkin diagram associated with a generalised Cartan matrix(GCM).

The Kac-Moody superalgebra $\mathcal{G}(A, \tau)$ is associated with a Dynkin diagram according to the following rules. Taking the assumption that $i \in \tau$ if $a_{ii} = 0$.

From a GCM A with each i of the diagonal entries (a_{ii}) 2 and $i \notin \tau$ a white dot and $i \in \tau$ a black dot $\bullet$, to each i such that $a_{ii} = 0$ and $i \in \tau$ a grey dot $\otimes$. The i-th and j-th roots will be joined by $\zeta_{ij} = \max\Big(|a_{ij}|, |a_{ji}|\Big)$ lines with $|a_{ij}\, a_{ji}| \leq 4$ and the off digonal entries nonzero where for off diagonal entries zero; then the number of connection lines are $|a_{ij}| = |a_{ji}|$ with $|a_{ij}|$ and $|a_{ji}| \leq 4$

The arrows will be added on the lines connecting the i-th and j-th dots when $\zeta_{ij} > 1$ and $|a_{ij}| \neq |a_{ji}|$, pointing from j to i if $|a_{ij}| > 1$. One can get the different Dynkin diagrams with details in [6, 5, 8].

5. A Realization of Affine Kac-Moody superalgebras

Let $L = \mathbb{C}[t, t^1]$ be an algebra of Laurent polynomial in t. The residue of a Laurent polynomial $P = \sum_{k \in \mathbb{Z}} c_k^k$ (where all but a finite number of c_k are 0) is defined as $ResP = c_{-1}$. Let $\mathcal{G}$ be a simple Lie superalgebra. Let $\mathcal{G}$ be a finite dimensional simple Lie superalgebra ($\mathcal{G} \neq gl(n|n)$,), $(.|.)$ be a nondegenerate invariant symmetric bilinear form on $\mathcal{G}$. The definition of affine untwisted B.S.A. $\mathcal{G}^{(1)}$ follows that of affine algebras, i.e. $\mathcal{G}^{(1)}$ is the loop algebra constructed from $\mathcal{G}$. Define an infinite dimensional superalgebra $\mathcal{G}^{(1)}$ as $\mathcal{G} \otimes \mathbb{C}[t, t^{-1}] \oplus \mathbb{C}D \oplus \mathbb{C}K$ here D, K are even elements and bracket is defined by

$$[X \otimes t^k, Y \otimes t^l] = [X, Y] \otimes t^{k+l} + k\delta_{k,-l}(X, Y)K,$$

$$[D, K] = 0$$

,

$$[D, X \otimes t^k] = kX \otimes t^k$$

Untwisted Affine B.S.A. Properties on the structure of affine Lie superalgebras can also be deduced by extending the classification of Dynkin diagrams to the affine case. This will in particular allow us to construct in a diagrammatic way twisted affine superalgebras from untwisted ones.

A simple root system of an affine B.S.A. $\mathcal{G}^{(1)}$ is obtained from a simple root system B of $\mathcal{G}$ by adding to it the affine root which project on B as the corresponding lowest root. The simple root systems of $\mathcal{G}^{(1)}$ are therefore associated to the extended Dynkin diagrams used to determine the regular subsuperalgebras.

6. Root of $\mathcal{G}^1$

(i) $A^{(1)}(m,n) = spl^{(1)}(m+1, n+1)$
$\Phi \cup \phi = \{\alpha_0 = k + \delta_{n+1} - e_1, \alpha_1 = e_1 - e_2, \cdots, \alpha_m = e_m - e_{m+1}, \alpha_{m+1} = e_{m+1} - \delta_1, \alpha_{m+2} = \delta_1 - \delta_2, \cdots, \alpha_{n+m+1} = \delta_n - \delta_{n+1}\}$

(ii) $B^{(1)}(m,n) = osp^{(1)}(2m+1, 2n)(m > 2)$
$\Phi \cup \phi = \{k - 2\delta_1, \alpha_1 = \delta_1 - \delta_2, \alpha_2 = \delta_2 - \delta_3, \cdots, \alpha_n = \delta_n - e_1, \alpha_{n+1} = e_1 - e_2, \alpha_{n+m+1} = e_{m-1} - e_m, \alpha_{n+m} = e_m$

(iii) $D^{(1)}(m+n) = osp^{(1)}(2m, 2n)(m > 2)$
$\Phi \cup \phi = \{k - 2\delta_1, \alpha_1 = \delta_1 - \delta_2, \alpha_2 = \delta_2 - \delta_3, \cdots, \alpha_n = \delta_n - e_1, \alpha_{n+1} = e_1 - e_2, \alpha_{n+m-1} = e_{m-1} - e_m, \alpha_{n+m-1} = e_{m-1} + e_m$

(iv) $C^{(1)}(n)$
$\Phi \cup \phi = \{\alpha_0 = k - e - \delta_1, \alpha_1 = e - \delta_1, \alpha_2 = \delta_1 - \delta_2, \cdots \alpha_n = \delta_{n-1} - \delta_n, \alpha_{n+1} = 2\delta_{n-1}\}$

(v) $D^{(1)}(2,1,\alpha)$
$\Phi \cup \phi = \{\alpha_0 = k - (e_1 + e_2 + e_3), e_1 - e_2 - e_3, 2e_2, 2e_3\}$

(vi) $F^{(1)}(4)$
$\Phi \cup \phi = \{\alpha_0 = k - 3\delta, \delta + \frac{1}{2}(-e_1 - e_2 - e_3), e_3, e_2 - e_3, e_1 - e_2)\}$

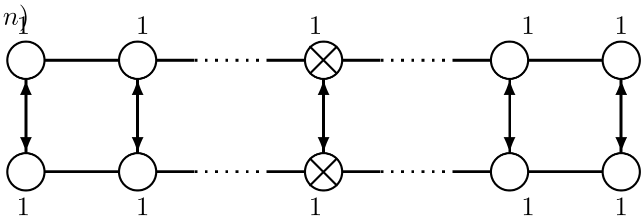
(vii) $G^{(1)}(3)$
$\Phi \cup \phi = \{\alpha_0 = k - 4\delta, \delta + e_1, e_2, e_3 - e_2\}$

The Cartan subalgebra of $\mathcal{G}^1$ is

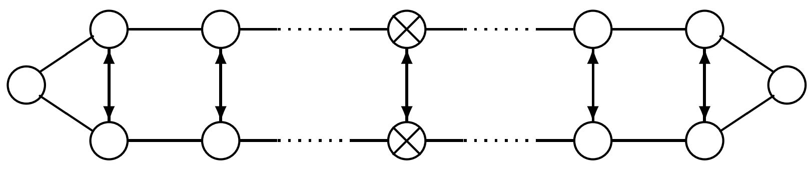
$$\mathfrak{h} = \mathring{\mathfrak{h}} \oplus \mathbb{C}K \oplus \mathbb{C}D$$

7. Real forms from Vogan diagram of affine untwisted Kac-Moody superalgebras

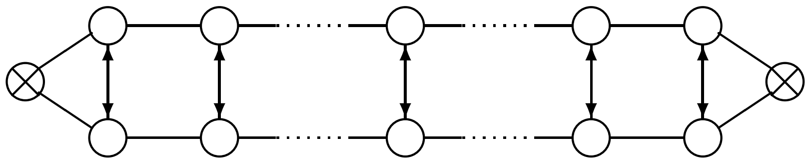
$A^{(1)}(m,n)$



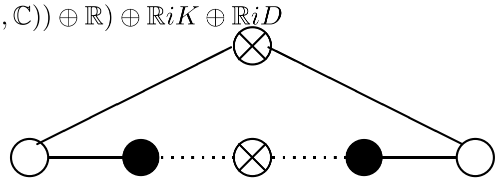
$L(t, t^1) \otimes (sl(m, \mathbb{R}) \oplus sl(n, \mathbb{R}) \oplus \mathbb{R}) \oplus \mathbb{R}iK \oplus \mathbb{R}iD$



$L(t, t^1) \otimes (su*(m) \oplus su*(n, \mathbb{R}) \oplus \mathbb{R}) \oplus \mathbb{R}iK \oplus \mathbb{R}iD$



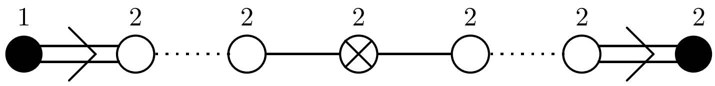
$L(t,t^1) \otimes (sl(n,\mathbb{C})) \oplus \mathbb{R}) \oplus \mathbb{R}iK \oplus \mathbb{R}iD$



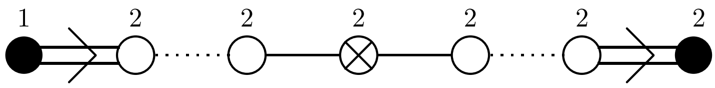
$L(t,t^{-1}) \otimes (su(p,m-p,\mathbb{R}) \oplus su(r,n-r,\mathbb{R}) \oplus i\mathbb{R}) \oplus \mathbb{R}ic \oplus \mathbb{R}id$

Case $B^{(1)}(m,n) = Osp^{(1)}(2m+1,2n)$

The below first Vogan diagram which contains the extreme right black painted root is from the original Dynkin diagram color.

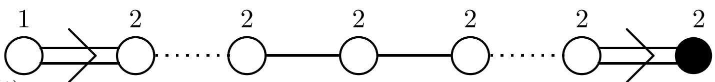


$L(t,t^{-1}) \otimes (sp(m,\mathbb{R}) \oplus \mathbb{R}ic \oplus \mathbb{R}id$

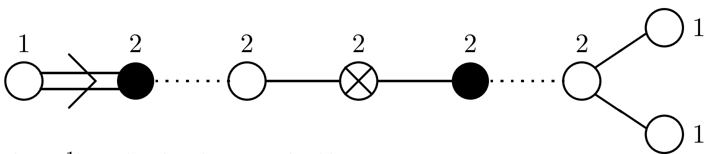


$L(t,t^{-1}) \otimes (sp(m,\mathbb{R}) \oplus so(p,q)) \oplus \mathbb{R}ic \oplus \mathbb{R}id$

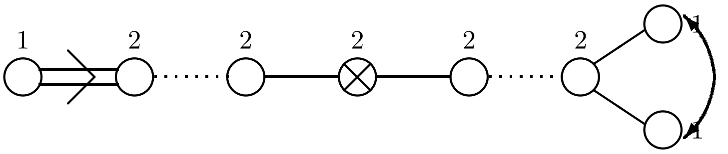
Case $B^{(1)}(0,n) = Osp^{(1)}(1,2n)$

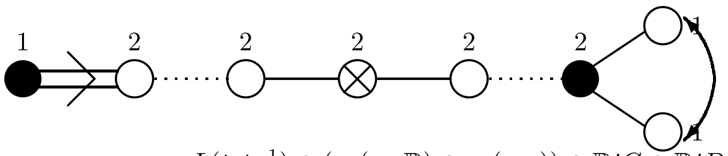


Case $D^{(1)}(m,n)$



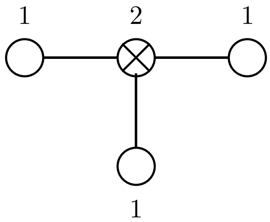
$L(t,t^{-1}) \otimes (sp(r,s) \oplus so^*(2p)) \oplus \mathbb{R}iC \oplus \mathbb{R}iD$



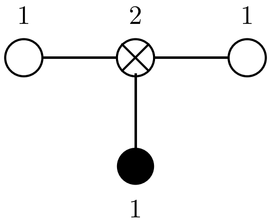


$L(t,t^{-1}) \otimes (sp(m,\mathbb{R}) \oplus so(p,q)) \oplus \mathbb{R}iC \oplus \mathbb{R}iD$

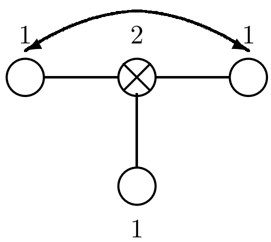
Real forms of $D^{(1)}(2,1;\alpha)$



$sl(2,\mathbb{R}) \oplus sl(2,\mathbb{R}) \oplus sl(2,\mathbb{R})$

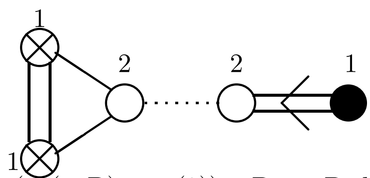


$su(2) \oplus su(2) \oplus sl(2,\mathbb{R})$

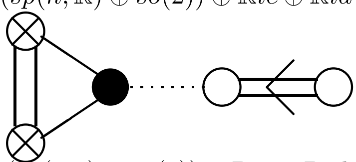


$sl(2,\mathbb{C} \oplus sl(2,\mathbb{R})$

Real forms of $C^{(1)}n$

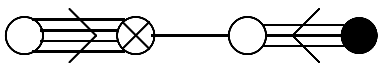


$L(t,t^{-1}) \otimes (sp(n,\mathbb{R}) \oplus so(2)) \oplus \mathbb{R}ic \oplus \mathbb{R}id$

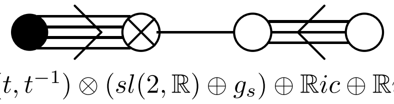


$L(t,t^{-1}) \otimes (sp(r,s) \oplus so(2)) \oplus \mathbb{R}ic \oplus \mathbb{R}id$

Real forms of $F^{(1)}(4)$

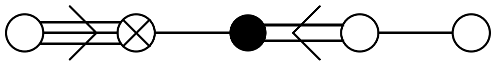


$L(t,t^{-1}) \otimes (sl(2,\mathbb{R}) \oplus g_c) \oplus \mathbb{R}ic \oplus \mathbb{R}id$

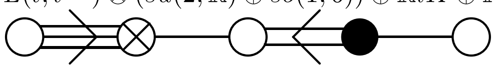


$L(t,t^{-1}) \otimes (sl(2,\mathbb{R}) \oplus g_s) \oplus \mathbb{R}ic \oplus \mathbb{R}id$

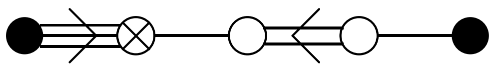
Real forms of $G^{(1)}(3)$



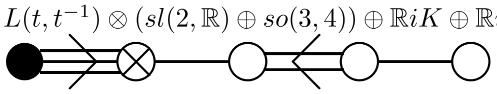
$L(t,t^{-1}) \otimes (su(2,\mathbb{R}) \oplus so(1,6)) \oplus \mathbb{R}iK \oplus \mathbb{R}iD$



$L(t,t^{-1}) \otimes (su(2,\mathbb{R}) \oplus so(2,5)) \oplus \mathbb{R}iK \oplus \mathbb{R}iD$



$L(t,t^{-1}) \otimes (sl(2,\mathbb{R}) \oplus so(3,4)) \oplus \mathbb{R}iK \oplus \mathbb{R}iD$



$L(t,t^{-1}) \otimes (sl(2,\mathbb{R}) \oplus so(7)) \oplus \mathbb{R}iK \oplus \mathbb{R}iD$

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References

[1] Batra P, *Vogan diagrams of affine Kac-Moody algebras*, Journal of Algebra 251, 80-97 (2002).
[2] Batra P, *Invariant of Real forms of Affine Kac-Moody Lie algebras*, Journal of Algebra 223,208-236 (2000).
[3] Chuah Meng-Kiat, *Cartan automorphisms and Vogan superdiagrams*,Math.Z.DOI 10.1007/s00209-012-1030-z.
[4] Chuah Meng-Kiat, *Finite order automorphism on contragredient Lie superalgebras*, Journal of Algebra 351 (2012) 138-159.
[5] Frappat L, Sciarrino A, *Hyperbolic Kac-Moody superalgebras*, arXiv:math-ph/0409041v1.
[6] Frappat L, Sciarrino A and Sorba, *Structure of basic Lie superalgebras and of their affine extensions*, commun. Math. Phys. 121,457-500 (1989)
[7] Kac V.G., *Infinite dimensional Lie algebras*, third edition 2003.
[8] Kac V.G.,*Lie superalgebras*, Adv. Math. 26,8 (1977).
[9] Parker M, *Classification of real simple Lie superalgebras of classical type*, J. Math.Phys. 21(4), April 1980.
[10] Ransingh B and Pati K C, *Vogan diagrams of Basic Lie superalgebra*, arXiv:1205.1394v1 [math.RT] 7 May 2012.
[11] Ray Urmie, *A character formula for generalized Kac-Moody superalgebras*, Journal of algebra 177, 154-163
[12] Knapp A.W., *Lie groups beyond an Introduction*, Second Edition.
[13] Pierre Henry-Labord, ab Bernard Juliab and Louis Paulotb, *Real borcherds superalgebras and M-theory* JHEP 04 (2003) 060.